Higgs boson, renormalization group, and naturalness in cosmology

A.O. Barvinsky[a], A.Yu. Kamenshchik[b,c], C. Kiefer[e], A.A. Starobinsky[c,d], C.F. Steinwachs[e]

[a] *Theory Department, Lebedev Physics Institute, Leninsky Prospect 53, Moscow 119991, Russia*
[b] *Dipartimento di Fisica and INFN, via Irnerio 46, 40126 Bologna, Italy*
[c] *L.D. Landau Institute for Theoretical Physics, Moscow 119334, Russia*
[d] *RESCEU, Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan*
[e] *Institut für Theoretische Physik, Universität zu Köln, Zülpicher Strasse 77, 50937 Köln, Germany*

Abstract

We consider the renormalization group improvement in the theory of the Standard Model (SM) Higgs boson playing the role of an inflaton with a strong non-minimal coupling to gravity. It suggests the range of the Higgs mass 135.6 GeV $\lesssim M_H \lesssim$ 184.5 GeV entirely determined by the lower WMAP bound on the CMB spectral index. We find the phenomenon of asymptotic freedom induced by this non-minimal curvature coupling, which brings the theory to the weak coupling domain. Asymptotic freedom fails at the boundaries of this domain, which makes the SM phenomenology sensitive to the current cosmological data and thus suggests future more precise CMB measurements as a SM test complementary to the LHC program. By using a concept of field-dependent cutoff we also show naturalness of the gradient and curvature expansion in this model within a conventional perturbation theory range of SM.

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1. Introduction

The hope for the forthcoming discovery of the Higgs boson at LHC revives the attempts of constructing a particle model accounting also for an inflationary scenario and its observable CMB spectrum. An obvious rationale behind this is the anticipation that cosmological observations can comprise Standard Model (SM) tests complimentary to collider experiments. While the relatively old work

Email addresses: barvin@td.lpi.ru (A.O. Barvinsky), kamenshchik@bo.infn.it (A.Yu. Kamenshchik), kiefer@thp.uni-koeln.de (C. Kiefer), alstar@landau.ac.ru (A.A. Starobinsky), cst@thp.uni-koeln.de (C.F. Steinwachs)

[1] had suggested that due to quantum effects inflation depends not only on the inflaton-graviton sector of the system but rather is strongly effected by the GUT contents of the particle model, the series of papers [2, 3, 4, 5, 6] transcended this idea to the SM ground with the Higgs field playing the role of an inflaton. This has recovered interest in a once rather popular [7, 1, 8, 9] model with the Lagrangian of the graviton-inflaton sector

$$\boldsymbol{L}(g_{\mu\nu},\varPhi) = \frac{1}{2}\left(M_P^2 + \xi|\varPhi|^2\right)R - \frac{1}{2}|\nabla\varPhi|^2 - V(|\Phi|), \tag{1}$$

$$V(|\Phi|) = \frac{\lambda}{4}(|\varPhi|^2 - v^2)^2, \quad |\varPhi|^2 = \varPhi^\dagger\varPhi, \tag{2}$$

where $\varPhi$ is a scalar field whose expectation value plays the role of an inflaton and which has a strong non-minimal curvature coupling with $\xi \gg 1$. Here, $M_P = m_P/\sqrt{8\pi} \approx 2.4 \times 10^{18}$ GeV is a reduced Planck mass, λ is a quartic self-coupling of $\varPhi$, and v is a symmetry breaking scale.

The motivation for this model was based on the observation [7] that the problem of an exceedingly small quartic coupling $\lambda \sim 10^{-13}$, dictated by the amplitude of primordial CMB perturbations [10], can be solved by using a non-minimally coupled inflaton with a large value of ξ. Later this model with the GUT-type sector of matter fields was used to generate initial conditions for inflation [1] within the concept of the no-boundary [11] and tunneling cosmological state [12]. The quantum evolution with these initial data was considered in [8, 13]. There it was shown that quantum effects are critically important for this scenario.

A similar theory but with the SM Higgs boson $\varPhi$ playing the role of an inflaton instead of the abstract GUT setup of [1, 8] was suggested in [2]. This work advocated the consistency of the corresponding CMB data with WMAP observations at the tree-level approximation of the theory, which was extended in [3] to the one-loop level. This has led to the lower bound on the Higgs mass $M_H \gtrsim 230$ GeV, originating from the observational restrictions on the CMB spectral index [3]. However, this conclusion which contradicts the widely accepted range 115 GeV$\leq M_H \leq 180$ GeV did not take into account $O(1)$ contributions due to renormalization group (RG) running, which qualitatively changes the situation. This was nearly simultaneously observed in [14, 4, 5] where the RG improvement of the one-loop results of [3] predicted the Higgs mass range almost coinciding with the conventional one.

Here we suggest the RG improvement of our one-loop results in [3] and find the CMB compatible range of the Higgs mass, both boundaries of this range being determined by the lower WMAP bound on the CMB spectral index, $n_s \gtrsim 0.94$ rather than by perturbation theory arguments. This makes the phenomenology of this gravitating SM essentially more sensitive to the cosmological bounds than in [4, 5, 6] and makes it testable by the CMB observations. The mechanism underlying these conclusions and explaining the efficiency of the perturbation theory at the inflationary scale is a phenomenon analogous to asymptotic freedom. Below we briefly present these results, whereas their detailed derivation and the discussion of their limitations can be found in [15].

2. One-loop approximation: CMB parameters in the theory with a large non-minimal curvature coupling

The usual understanding of non-renormalizable theories is that renormalization of higher-dimensional operators does not effect the renormalizable sector of low-dimensional operators, because the former ones are suppressed by powers of a cutoff – the Planck mass M_P [16]. Therefore, beta functions of the Standard Model sector are not expected to be modified by gravitons. The situation with the non-minimal coupling is more subtle. Due to mixing of the Higgs scalar field with the longitudinal part of gravity in the kinetic term of the Lagrangian (1), an obvious suppression of pure graviton loops by the effective Planck mass, $M_P^2+\xi\varphi^2 \gg M_P^2$, for large ξ proliferates to the sector of the Higgs field, so that certain parts of beta functions are strongly damped by large ξ [5]. Therefore, a special combination of coupling constants $\boldsymbol{A}$ which we call *anomalous scaling* [1] becomes very small and brings down the CMB compatible Higgs mass bound. The importance of this quantity follows from the fact observed in [1, 8, 3] that due to large ξ, quantum effects and their CMB manifestation are universally determined by $\boldsymbol{A}$. The nature of this quantity is as follows.

Let the model contain in addition to (1) also a set of scalar fields χ, vector gauge bosons A_μ and spinors ψ, which have an interaction with $\varPhi$ dictated by the local gauge invariance. If we denote by φ the inflaton – the only nonzero component of the mean value of $\varPhi$ in the cosmological state –, then the quantum effective action of the system takes a generic form

$$S[g_{\mu\nu},\varphi] = \int d^4x\, g^{1/2} \left(-V(\varphi) + U(\varphi)\, R(g_{\mu\nu}) - \frac{1}{2}\, G(\varphi)\, (\nabla\varphi)^2 + \ldots \right), \quad (3)$$

where $V(\varphi)$, $U(\varphi)$ and $G(\varphi)$ are the coefficients of the derivative expansion, and we disregard the contribution of higher-derivative operators negligible in the slow-roll approximation of the inflation theory. In this approximation the dominant quantum contribution to these coefficients comes from the heavy massive sector of the model. In particular, the masses of the physical particles and Goldstone modes $m(\varphi)$, generated by their quartic, gauge and Yukawa couplings with φ, give rise to the Coleman-Weinberg potential – the one-loop contribution to the effective potential V in (3). Since $m(\varphi) \sim \varphi$, for large φ this potential reads

$$V^{1-\mathrm{loop}}(\varphi) = \sum_{\mathrm{particles}} (\pm 1)\, \frac{m^4(\varphi)}{64\pi^2}\, \ln\frac{m^2(\varphi)}{\mu^2} = \frac{\lambda \boldsymbol{A}}{128\pi^2}\, \varphi^4 \ln\frac{\varphi^2}{\mu^2} + \ldots \quad (4)$$

and, thus, determines the dimensionless coefficient $\boldsymbol{A}$ – the anomalous scaling associated with the normalization scale μ in (4). Moreover, for $\xi \gg 1$ mainly this quantity and the dominant quantum correction to $U(\varphi)$ [15],

$$U^{1-\mathrm{loop}}(\varphi) = \frac{3\xi\lambda}{32\pi^2}\, \varphi^2 \ln\frac{\varphi^2}{\mu^2} + \ldots, \quad (5)$$

determine the quantum rolling force in the effective equation of the inflationary dynamics [8, 13] and yields the parameters of the CMB generated during inflation [3].

Inflation and its CMB are easy to analyze in the Einstein frame of fields $\hat{g}_{\mu\nu}$, $\hat{\varphi}$ in which the action $\hat{S}[\hat{g}_{\mu\nu}, \hat{\varphi}] = S[g_{\mu\nu}, \varphi]$ has a minimal coupling, $\hat{U} = M_P^2/2$, canonically normalized inflaton field, $\hat{G} = 1$, and the new inflaton potential $\hat{V} = M_P^4 V(\varphi)/4U^2(\varphi)$.[1] At the inflationary scale with $\varphi > M_P/\sqrt{\xi} \gg v$ and $\xi \gg 1$, this potential reads

$$\hat{V} = \frac{\lambda M_P^4}{4\,\xi^2}\left(1 - \frac{2M_P^2}{\xi\varphi^2} + \frac{\boldsymbol{A_I}}{16\pi^2}\ln\frac{\varphi}{\mu}\right), \tag{6}$$

where the parameter $\boldsymbol{A_I}$ represents the anomalous scaling (14) modified by the quantum correction to the non-minimal curvature coupling (5),

$$\boldsymbol{A_I} = \boldsymbol{A} - 12\lambda = \frac{3}{8\lambda}\Big(2g^4 + \big(g^2 + g'^2\big)^2 - 16y_t^4\Big) - 6\lambda. \tag{7}$$

This quantity – which we will call *inflationary anomalous scaling* – enters the expressions for the slow-roll parameters, $\hat{\varepsilon} \equiv (M_P^2/2V^2)(d\hat{V}/d\hat{\varphi})^2$ and $\hat{\eta} \equiv (M_P^2/\hat{V})d^2\hat{V}/d\hat{\varphi}^2$, and ultimately determines all the inflation characteristics. In particular, smallness of $\hat{\varepsilon}$ yields the range of the inflationary stage $\varphi > \varphi_{\rm end}$, terminating at the value of $\hat{\varepsilon}$, which we chose to be $\hat{\varepsilon}_{\rm end} = 3/4$. Then the inflaton value at the exit from inflation equals $\varphi_{\rm end} \simeq 2M_P/\sqrt{3\xi}$ under the natural assumption that perturbation expansion is applicable for $\boldsymbol{A_I}/64\pi^2 \ll 1$. The value of φ at the beginning of the inflation stage of duration N in units of the e-folding number then reads [3]

$$\varphi^2 = \frac{4N}{3}\frac{M_P^2}{\xi}\frac{e^x - 1}{x}, \tag{8}$$

$$x \equiv \frac{N\boldsymbol{A_I}}{48\pi^2}, \tag{9}$$

where a special parameter x directly involves the anomalous scaling $\boldsymbol{A_I}$.

This relation determines the Fourier power spectrum for the scalar metric perturbation ζ, $\Delta_\zeta^2(k) \equiv < k^3\zeta_{\mathbf{k}}^2 >= \hat{V}/24\pi^2 M_P^4\hat{\varepsilon}$, where the right-hand side is taken at the first horizon crossing, $k = aH$, relating the comoving perturbation wavelength k^{-1} to the e-folding number N,

$$\Delta_\zeta^2(k) = \frac{N^2}{72\pi^2}\frac{\lambda}{\xi^2}\left(\frac{e^x - 1}{x\,e^x}\right)^2. \tag{10}$$

The CMB spectral index $n_s \equiv 1 + d\ln\Delta_\zeta^2/d\ln k = 1 - 6\hat{\varepsilon} + 2\hat{\eta}$ and the tensor

[1] The Einstein and Jordan frames are related by the equations $\hat{g}_{\mu\nu} = 2U(\varphi)g_{\mu\nu}/M_P^2$, $(d\hat{\varphi}/d\varphi)^2 = M_P^2(GU + 3U'^2)/2U^2$.

to scalar ratio $r = 16\hat{\varepsilon}$ correspondingly read as[2]

$$n_s = 1 - \frac{2}{N}\,\frac{x}{e^x - 1}\;, \tag{11}$$

$$r = \frac{12}{N^2}\left(\frac{xe^x}{e^x - 1}\right)^2\;. \tag{12}$$

Therefore, with the spectral index constraint $0.94 < n_s(k_0) < 0.99$ (the combined WMAP+BAO+SN data at the pivot point $k_0 = 0.002\,\mathrm{Mpc}^{-1}$ corresponding to $N \simeq 60$ [18]) these relations immediately give the range of anomalous scaling $-12 < \boldsymbol{A_I} < 14$ [3].

On the other hand, in the Standard Model $\boldsymbol{A}$ is expressed in terms of the masses of the heaviest particles – $W^\pm$ boson, Z boson and top quark,

$$m_W^2 = \frac{1}{4}\,g^2\,\varphi^2,\;\; m_Z^2 = \frac{1}{4}\,(g^2 + g'^2)\,\varphi^2,\;\; m_t^2 = \frac{1}{2}\,y_t^2\,\varphi^2, \tag{13}$$

and the mass of three Goldstone modes $m_G^2 = V'(\varphi)/\varphi = \lambda(\varphi^2 - v^2) \simeq \lambda\varphi^2$. Here, g and g' are the $SU(2) \times U(1)$ gauge couplings, g_s is the $SU(3)$ strong coupling, and y_t is the Yukawa coupling for the top quark. At the inflation stage the Goldstone mass m_G^2 is non-vanishing in contrast to its zero on-shell value in the electroweak vacuum [19]. Therefore, Eq. (4) gives the expression

$$\boldsymbol{A} = \frac{3}{8\lambda}\Big(2g^4 + \big(g^2 + g'^2\big)^2 - 16y_t^4\Big) + 6\lambda. \tag{14}$$

In the conventional range of the Higgs mass 115 GeV$\leq M_H \leq$ 180 GeV [20] this quantity at the electroweak scale belongs to the range $-48 < \boldsymbol{A} < -20$ which strongly contradicts the CMB range given above.

However, the RG running of coupling constants is strong enough and drives $\boldsymbol{A}$ to the CMB compatible range at the inflation scale. Below we show that the formalism of [3] stays applicable but with the electroweak $\boldsymbol{A}$ replaced by the running $\boldsymbol{A}(t)$, where $t = \ln(\varphi/\mu)$ is the running scale of the renormalization group (RG) improvement of the effective potential [21].

3. RG improvement

According to the Coleman-Weinberg technique [21] the one-loop RG improved effective action has the form (3) with

$$V(\varphi) = \frac{\lambda(t)}{4}\,Z^4(t)\,\varphi^4, \tag{15}$$

$$U(\varphi) = \frac{1}{2}\Big(M_P^2 + \xi(t)\,Z^2(t)\,\varphi^2\Big), \tag{16}$$

$$G(\varphi) = Z^2(t). \tag{17}$$

[2]Note that for $|x| \ll 1$ these predictions exactly coincide with those [23] of the $f(R) = M_P^2(R + R^2/6M^2)/2$ inflationary model [24] with the scalar particle (scalaron) mass $M = M_P\sqrt{\lambda}/\sqrt{3}\xi$.

Here the running scale $t = \ln(\varphi/M_t)$ is normalized at the top quark mass $\mu = M_t$ (we denote physical (pole) masses by capital letters in contrast to running masses (13) above) [3]. The running couplings $\lambda(t)$, $\xi(t)$ and the field renormalization $Z(t)$ incorporate summation of powers of logarithms and belong to the solution of the RG equations

$$\frac{dg_i}{dt} = \beta_{g_i}, \quad \frac{dZ}{dt} = \gamma Z \tag{18}$$

for the full set of coupling constants $g_i = (\lambda, \xi, g, g', g_s, y_t)$ in the "heavy" sector of the model with the corresponding beta functions β_{g_i} and the anomalous dimension γ of the Higgs field.

An important subtlety with these β functions is the effect of non-minimal curvature coupling of the Higgs field. For large ξ the kinetic term of the tree-level action has a strong mixing between the graviton $h_{\mu\nu}$ and the quantum part of the Higgs field σ on the background φ. Symbolically it has the structure

$$(M_P^2 + \xi^2\varphi^2)h\nabla\nabla h + \xi\varphi\sigma\nabla\nabla h + \sigma\Box\sigma,$$

which yields a propagator whose elements are suppressed by a small $1/\xi$-factor in all blocks of the 2×2 graviton-Higgs sector. For large $\varphi \gg M_P/\sqrt{\xi}$, the suppression of pure graviton loops is, of course, obvious because of the effective Planck mass squared essentially exceeding the Einstein one, $M_P^2 + \xi\varphi^2 \gg M_P^2$. Due to mixing, this suppression proliferates to the full graviton-Higgs sector of the theory and gives the Higgs propagator $s(\varphi)/(\Box - m_H^2)$ weighted by the suppression factor $s(\varphi)$

$$s(\varphi) = \frac{M_P^2 + \xi\varphi^2}{M_P^2 + (6\xi + 1)\xi\varphi^2}. \tag{19}$$

This mechanism [25, 8, 13] modifies the beta functions of the SM sector [5] at high energy scales because the factor $s(\varphi)$, which is close to one at the EW scale $v \ll M_P/\xi$, is very small for $\varphi \gg M_P/\sqrt{\xi}$, $s \simeq 1/6\xi$. Such a modification, in fact, justifies the extension beyond the scale M_P/ξ interpreted in [26, 27] as a natural validity cutoff of the theory[4].

There is an important subtlety with the modification of beta functions which was disregarded in [5] (and in the first version of [15]). Goldstone modes, in

[3] Application of the Coleman-Weinberg technique removes the ambiguity in the choice of the RG scale in cosmology – an issue discussed in [22].

[4] The smallness of this cutoff could be interpreted as inefficiency of the RG analysis beyond the range of trustability of the model. However, the cutoff $M_P/\xi \ll M_P$ of [26, 27] applies to energies (momenta) of scattering processes in flat spacetime with a small EW value of φ. For the inflation stage on the background of a large φ this cutoff gets modified due to the increase in the effective Planck mass $M_P^2 + \xi\varphi^2 \gg M_P^2$ (and the associated decrease of the s-factor (19) – resummation of terms treated otherwise as perturbations in [26]). Thus the magnitude of the Higgs field at inflation is not really indicative of the violation of the physical cutoff bound (see discussion in Sects. 5 and 6 below).

contrast to the Higgs particle, do not have a kinetic term mixing with gravitons [6]. Therefore, their contribution is not suppressed by the s-factor of the above type. Separation of Goldstone contributions from the Higgs ones leads to the following modification of the one-loop beta functions, which is essentially different from that of [5] (cf. also [29])

$$\beta_\lambda = \frac{\lambda}{16\pi^2}\left(18s^2\lambda + \boldsymbol{A}(t)\right) - 4\gamma\lambda, \tag{20}$$

$$\beta_\xi = \frac{6\xi}{16\pi^2}(1+s^2)\lambda - 2\gamma\xi, \tag{21}$$

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\left(-\frac{2}{3}g'^2 - 8g_s^2 + \left(1+\frac{s}{2}\right)y_t^2\right) - \gamma y_t, \tag{22}$$

$$\beta_g = -\frac{39-s}{12}\frac{g^3}{16\pi^2}, \tag{23}$$

$$\beta_{g'} = \frac{81+s}{12}\frac{g'^3}{16\pi^2}, \tag{24}$$

$$\beta_{g_s} = -\frac{7g_s^3}{16\pi^2}. \tag{25}$$

Here the anomalous dimension of the Higgs field γ is given by a standard expression in the Landau gauge

$$\gamma = \frac{1}{16\pi^2}\left(\frac{9g^2}{4} + \frac{3g'^2}{4} - 3y_t^2\right), \tag{26}$$

the anomalous scaling $\boldsymbol{A}(t)$ is defined by (14) and we retained only the leading terms in $\xi \gg 1$. It will be important in what follows that this anomalous scaling contains the Goldstone contribution 6λ, so that the full β_λ in (20) has a λ^2-term unsuppressed by $s(\varphi)$ at large scale $t = \ln(\varphi/\mu)$.

The inflationary stage in units of Higgs field e-foldings is very short, which allows one to use the approximation linear in $\Delta t \equiv t - t_{\rm end} = \ln(\varphi/\varphi_{\rm end})$, where the initial data point is chosen at the end of inflation $t_{\rm end}$. Therefore, for beta functions (20) and (21) with $s \ll 1$ we have

$$\lambda(t) = \lambda_{\rm end}\left(1 - 4\gamma_{\rm end}\Delta t + \frac{\boldsymbol{A}(t_{\rm end})}{16\pi^2}\,\Delta t\right), \tag{27}$$

$$\xi(t) = \xi_{\rm end}\Big(1 - 2\gamma_{\rm end}\Delta t + \frac{6\lambda_{\rm end}}{16\pi^2}\Delta t\Big), \tag{28}$$

where $\lambda_{\rm end}$, $\gamma_{\rm end}$, $\xi_{\rm end}$ are determined at $t_{\rm end}$ and $\boldsymbol{A}_{\rm end} = \boldsymbol{A}(t_{\rm end})$ is also a particular value of the running anomalous scaling (14) at the end of inflation.

On the other hand, the RG improvement of the effective action (15)-(17) implies that this action coincides with the tree-level action for a new field $\phi = Z(t)\varphi$ with running couplings as functions of $t = \ln(\varphi/\mu)$ (the running of $Z(t)$ is slow and affects only the multi-loop RG improvement). Then, in view of (15)-(16) the RG improved potential takes at the inflation stage the form of the one-loop potential (6) for the field ϕ with a particular choice of the normalization

point $\mu = \phi_{\rm end}$ and all the couplings replaced by their running values taken at $t_{\rm end}$. Therefore, the formalism of [3] can be directly applied to find the CMB parameters of the model, which now turn out to be determined by the running anomalous scaling $\boldsymbol{A_I}(t)$ taken at $t_{\rm end}$.

In contrast to the inflationary stage, the post-inflationary running is very large and requires numerical simulation. We fix the $t = 0$ initial conditions for the RG equations (18) at the top quark scale $M_t = 171$ GeV. For the constants g, g' and g_s, they read [20]

$$g^2(0) = 0.4202,\ g'^2(0) = 0.1291,\ g_s^2(0) = 1.3460, \tag{29}$$

where $g^2(0)$ and $g'^2(0)$ are obtained by a simple one-loop RG flow from the conventional values of $\alpha(M_Z) \equiv g^2/4\pi = 0.0338$, $\alpha'(M_Z) \equiv g'^2/4\pi = 0.0102$ at M_Z-scale, and the value $g_s^2(0)$ at M_t is generated by the numerical program of [30]. The analytical algorithm of transition between different scales for g_s^2 was presented in [31]. For the Higgs self-interaction constant λ and for the Yukawa top quark interaction constant y_t the initial conditions are determined by the pole mass matching scheme originally developed in [32] and presented in the Appendix of [33].

The initial condition $\xi(0)$ follows from the CMB normalization (10), $\Delta_\zeta^2 \simeq 2.5 \times 10^{-9}$ at the pivot point $k_0 = 0.002$ Mpc^{-1} [18] which we choose to correspond to $N \simeq 60$. This yields the following estimate on the ratio of coupling constants

$$\frac{1}{Z_{\rm in}^2}\frac{\lambda_{\rm in}}{\xi_{\rm in}^2} \simeq 0.5 \times 10^{-9} \left(\frac{x_{\rm in}\ \exp x_{\rm in}}{\exp x_{\rm in} - 1}\right)^2 \tag{30}$$

at the moment of the first horizon crossing for $N = 60$ which we call the "beginning" of inflation and label by $t_{\rm in} = \ln(\varphi_{\rm in}/M_t)$ with $\varphi_{\rm in}$ defined by (8). Thus, the RG equations (18) for six couplings $(g, g', g_s, y_t, \lambda, \xi)$ with five initial conditions and the final condition for ξ uniquely determine the needed RG flow.

The RG flow covers also the inflationary stage from the chronological end of inflation $t_{\rm end}$ to $t_{\rm in}$. At the end of inflation we choose the value of the slow roll parameter $\hat\varepsilon = 3/4$, and $\varphi_{\rm end} \equiv M_t e^{t_{\rm end}} \simeq M_P\sqrt{4/3\xi_{\rm end}}$. Thus, the duration of inflation in units of inflaton field e-foldings $t_{\rm in} - t_{\rm end} = \ln(\varphi_{\rm in}/\varphi_{\rm end}) \simeq \ln N/2 \sim 2$ [15] is very short relative to the post-inflationary evolution $t_{\rm end} \sim 35$. The approximation linear in logs implies the bound $|\boldsymbol{A_I}(t_{\rm end})|\Delta t/16\pi^2 \ll 1$, which in view of $\Delta t < t_{\rm in} - t_{\rm end} \simeq \ln N/2$ holds for $|\boldsymbol{A_I}(t_{\rm end})|/16\pi^2 \ll 0.5$.

4. Numerical analysis

The running of $\boldsymbol{A}(t)$ strongly depends on the behavior of $\lambda(t)$. For small Higgs masses the usual RG flow in SM leads to an instability of the EW vacuum caused by negative values of $\lambda(t)$ in a certain range of t [34, 33]. The same happens in the presence of non-minimal curvature coupling. The numerical solution for $\lambda(t)$ is shown in Fig.1 for five values of the Higgs mass and the

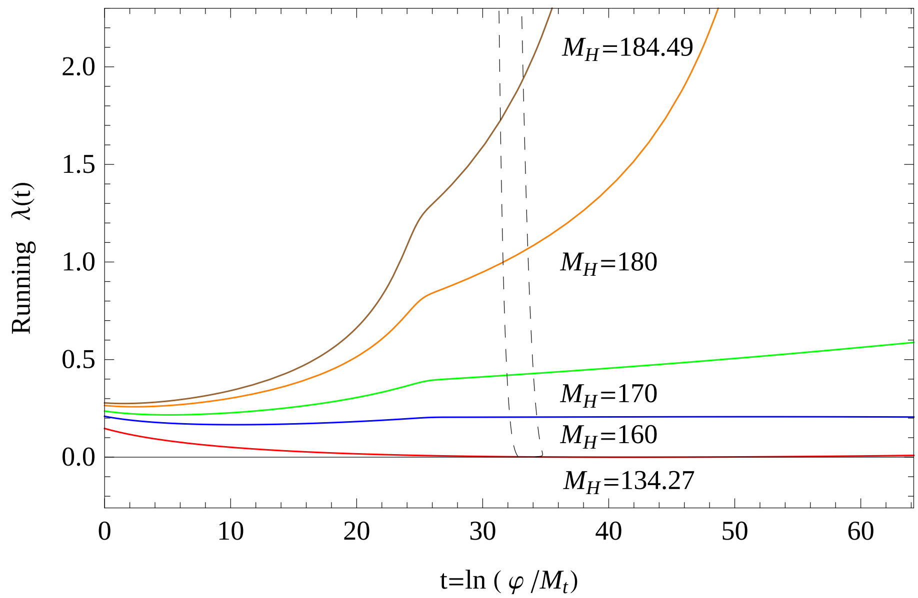

Figure 1: Running $\lambda(t)$ for five values of the Higgs mass above the instability threshold. Dashed curves mark the boundaries of the inflation domain $t_{\rm end} \leq t \leq t_{\rm in}$.

value of top quark mass $M_t = 171$ GeV. The lowest one corresponds to the boundary of the instability window,

$$M_H^{\rm inst} \simeq 134.27 \text{ GeV}, \tag{31}$$

for which $\lambda(t)$ bounces back to positive values after vanishing at $t_{\rm inst} \sim 41.6$ or $\varphi_{\rm inst} \sim 80 M_P$. It turns out that the corresponding $\xi(t)$ is nearly constant and is about 5000 (see below), so that the factor (19) at $t_{\rm inst}$ is very small $s \simeq 1/6\xi \sim 0.00005$. Thus the situation is different from the usual Standard Model with $s = 1$, and numerically the critical value turns out to be higher than the known SM stability bound ~ 125 GeV [33].

Fig.1 shows that near the instability threshold $M_H = M_H^{\rm inst}$ the running coupling $\lambda(t)$ stays very small for all scales t relevant to the observable CMB. This follows from the fact that the positive running of $\lambda(t)$ caused by the term $(18s^2 + 6)\lambda^2$ in β_λ, (20), is much slower for $s \ll 1$ than that of the usual SM driven by the term $24\lambda^2$.

For all Higgs masses in the range $M_H^{\rm inst} = 134.27 \text{ GeV} < M_H < 185$ GeV the inflation range $t_{\rm end} < t < t_{\rm in}$ is always below $t_{\rm inst} = 41.6$, so that from Fig. 2 $\boldsymbol{A_I}(t)$ is always negative during inflation. Its running explains the main difference from the results of one-loop calculations in [3]. $\boldsymbol{A_I}(t)$ runs from big negative values $\boldsymbol{A_I}(0) < -20$ at the electroweak scale to small also negative values at the inflation scale below $t_{\rm inst}$. This makes the CMB data compatible with the generally accepted Higgs mass range. Indeed, the knowledge of the

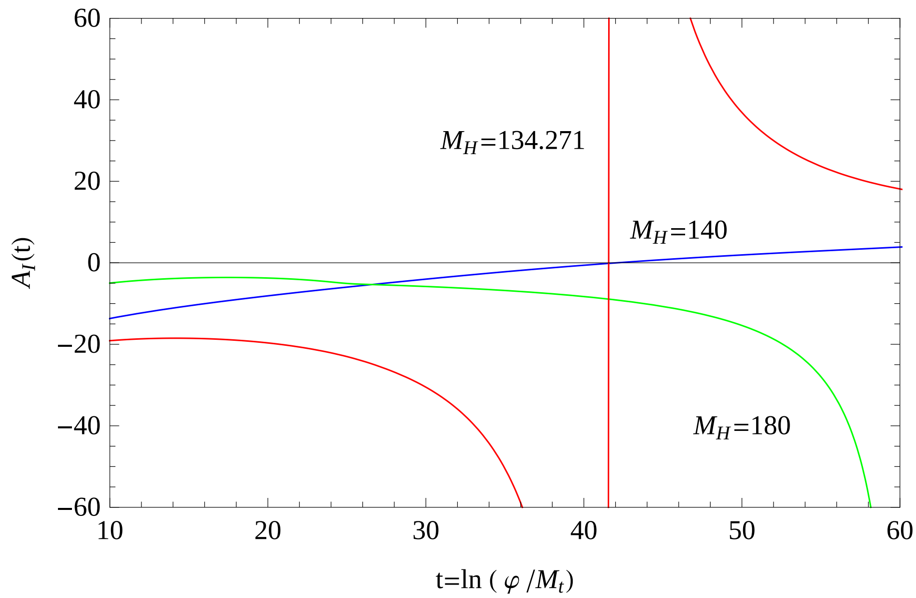

Figure 2: Running anomalous scaling for the critical Higgs mass (the red curve with a vertical segment at the singularity with $t_{\rm inst} \sim 41.6$) and for two masses in the stability domain (blue and green curves).

RG flow immediately allows one to obtain $\boldsymbol{A_I}(t_{\rm end})$ and $x_{\rm end}$ and thus find the parameters of the CMB power spectrum (11)-(12) as functions of M_H. The parameter of primary interest – spectral index – is given by Eq. (11) with $x = x_{\rm end} \equiv N\boldsymbol{A_I}(t_{\rm end})/48\pi^2$ and depicted in Fig.3. Even for low values of Higgs mass above the stability bound, n_s falls into the range admissible by the CMB constraint existing now at the 2σ confidence level (based on the combined WMAP+BAO+SN data [18]) $0.94 < n_s(k_0) < 0.99$.

The spectral index drops below 0.94 only for large $x_{\rm end} < 0$ or large negative $\boldsymbol{A_I}(t_{\rm end})$, which happens only when M_H either approaches the instability bound or exceeds 180 GeV at the decreasing branch of the n_s graph. Thus, we get the lower and upper bounds on the Higgs mass, which both follow from the lower bound of the CMB data. Numerical analysis for the corresponding $x_{\rm end} \simeq -1.4$ gives for $M_t = 171$ GeV the range of CMB compatible Higgs mass

$$135.62 \text{ GeV} \lesssim M_H \lesssim 184.49 \text{ GeV}. \tag{32}$$

Both bounds belong to the nonlinear domain of the equation (11) with $x_{\rm end} \simeq -1.4$, but the quantity $|\boldsymbol{A_I}(t_{\rm end})|/16\pi^2 = 0.07 \ll 0.5$ satisfies the restriction mentioned above, and their calculation is still in the domain of our linear in logs approximation.

The upper bound on n_s does not generate restrictions on M_H. The lower CMB bound in (32) is slightly higher than the instability bound $M_H^{\rm inst} = 134.27$

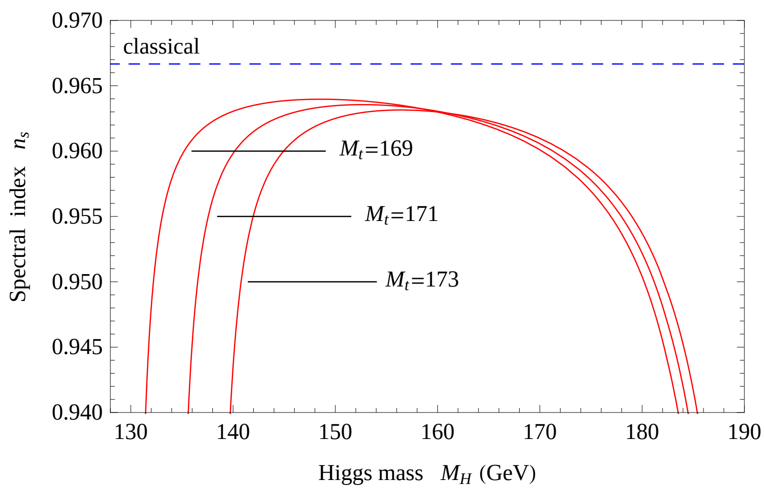

Figure 3: The spectral index n_s as a function of the Higgs mass M_H for three values of the top quark mass.

GeV. In turn, this bound depends on the initial data for weak and strong couplings and on the top quark mass M_t which is known with less precision. The above bounds were obtained for $M_t = 171$ GeV. Results for the neighboring values $M_t = 171 \pm 2$ GeV are presented in Fig. 3 to show the pattern of their dependence on M_t.

Finally let us focus on the running of $\xi(t)$ depicted for five values of the Higgs mass in Fig.4 starting with the lower bound of the range (37). It is very slow for low values of the Higgs mass near the instability threshold. Of course, this follows from the smallness of the running $\lambda(t)$ in this domain. Another property of the ξ-behavior is that the normalization of the power spectrum leads to a value $\xi \sim 5000$ for small Higgs masses, which is smaller than the old estimate $\sim 10^4$ [7, 1, 8, 13, 2, 3]. This is caused by a decrease of $\lambda(t)$ which at $t_{\rm in}$ becomes much smaller than $\lambda(0)$ [5].

5. Gradient and curvature expansion cutoff and naturalness

The expression (3) is a truncation of the curvature and derivative expansion of the full effective action. It was repeatedly claimed that with large ξ the weak field version of this expansion on flat (and empty) space background has a cutoff $4\pi M_P/\xi$ [26, 27]. This scale is essentially lower than the Higgs field during inflation $\varphi \sim M_P/\sqrt{\xi}$ and, therefore, seems to invalidate predictions based on (3) without unnatural suppression of higher-dimensional operators. The attempt to improve the situation by transition to the Einstein frame [35] was claimed to fail [36, 37, 38] in view of a multiplet nature of the Higgs field involving Nambu-Goldstone modes.

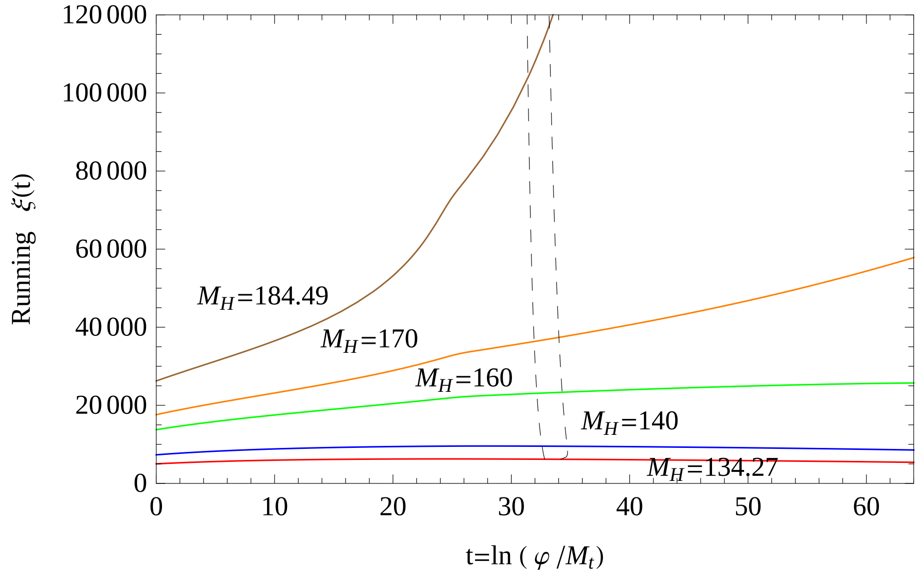

Figure 4: Plots of running $\xi(t)$.

Here we show that these objections against naturalness are not conclusive. First, as mentioned above, a big value of φ during inflation is not really indicative of a large physical scale of the problem. In contrast to curvature and energy density the inflaton itself is not a physical observable, but rather a configuration space coordinate of the model. Secondly, we show now that the inflation scale actually lies below the gradient expansion cutoff, and this justifies naturalness of the obtained results. No transition to another conformal frame is needed for that, but rather the resummation accounting for transition to large φ background.

Indeed, the main peculiarity of the model (1) is that in the background field method with small derivatives the role of the effective Planck mass is played by $\sqrt{M_P^2+\xi\varphi^2}$. The power-counting method of [26] underlying the derivation of the cutoff $4\pi M_P/\xi$ also applies here but with the Planck mass M_P replaced by the effective one, $M_P \to \sqrt{M_P^2+\xi\varphi^2} > \sqrt{\xi}\varphi$. The resulting cutoff is thus bounded from below by

$$\Lambda(\varphi) = \frac{4\pi\varphi}{\sqrt{\xi}}, \tag{33}$$

and this bound can be used as a *running* cutoff of the gradient and curvature expansion. The origin of this cutoff can be demonstrated in the one-loop approximation. When calculated in the *Jordan* frame, the one-loop divergences quadratic in the curvature R have a strongest in ξ contribution (this can be easily deduced from Appendix of [15])

$$\xi^2 \frac{R^2}{16\pi^2}. \tag{34}$$

As compared to the tree-level part linear in the curvature $\sim (M_P^2 + \xi\varphi^2)R$, the one loop R^2-term turns out to be suppressed by the above cutoff factor $16\pi^2(M_P^2 + \xi\varphi^2)/\xi^2 \simeq \Lambda^2$.

The on-shell curvature estimate at the inflation stage reads $R \sim V/U \sim \lambda\varphi^2/\xi$ in the Jordan frame, so that the resulting curvature expansion runs in powers of

$$\frac{R}{\Lambda^2} \sim \frac{\lambda}{16\pi^2} \tag{35}$$

and remains efficient in the usual perturbation theory range of SM, $\lambda/16\pi^2 \ll 1$. This works perfectly well in our Higgs inflation model, because in the full CMB compatible range of the Higgs mass $\lambda < 2$ (see Fig.1).

From the viewpoint of the gradient expansion for φ this cutoff is even more efficient. Indeed, the inflaton field gradient can be expressed in terms of the inflaton potential $\hat{V}$ and the inflation smallness parameter $\hat{\varepsilon}$ taken in the Einstein frame, $\dot{\varphi} \simeq (\varphi^2/M_P^2)(\xi\hat{\varepsilon}\hat{V}/18)^{1/2}$. With $\hat{V} \simeq \lambda M_P^4/4\xi^2$ this immediately yields the gradient expansion in powers of

$$\frac{\partial}{\Lambda} \sim \frac{1}{\Lambda}\frac{\dot{\varphi}}{\varphi} \simeq \frac{\sqrt{\lambda}}{48\pi}\sqrt{2\hat{\varepsilon}}, \tag{36}$$

which is even better than (35) by the factor ranging from $1/N$ at the beginning of inflation to $O(1)$ at the end of it.

Eqs. (35) and (36) justify the effective action truncation in (3) in the inflationary domain. Thus only multi-loop corrections to the coefficient functions $V(\varphi)$, $U(\varphi)$ and $G(\varphi)$ might stay beyond control in the form of higher-dimensional operators $(\varphi/\Lambda)^n$ and violate the flatness of the effective potential necessary for inflation. However, in view of the form of the running cutoff (33) they might be large, but do not affect the shape of these coefficient functions because of the field independence of the ratio φ/Λ. Only the logarithmic running of couplings in (15)-(17) controlled by RG dominates the quantum input in the inflationary dynamics and its CMB spectra[5].

6. Conclusions and discussion

We have found that the considered model looks remarkably consistent with CMB observations in the Higgs mass range

$$135.6 \text{ GeV} \lesssim M_H \lesssim 184.5 \text{ GeV}, \tag{37}$$

which is very close to the widely accepted range dictated by electroweak vacuum stability and perturbation theory bounds.

This result is based on the RG improvement of our analytical results in [3]. We have completely recovered the analytic formalism of [3] for all inflation

[5]Like the logarithmic term of (6) which dominates over the nearly flat classical part of the inflaton potential and qualitatively modifies tree-level predictions of the theory [3].

parameters, which only gets modified by the RG mapping between the coupling constants at the EW scale and those at the scale of inflation. A peculiarity of this formalism is that for large $\xi \gg 1$ the effect of SM phenomenology on inflation is universally encoded in one quantity – the anomalous scaling $\boldsymbol{A_I}$. It was earlier suggested in [1] for a generic gauge theory, and in SM it is dominated by contributions of heavy particles – ($W^{\pm}$, Z)-bosons, top quark and Goldstone modes. This quantity is forced to run in view of RG resummation of leading logarithms, and this running raises a large negative EW value of $\boldsymbol{A_I}$ to a *small negative* value at the inflation scale. Ultimately this leads to the admissible range of Higgs masses very close to the conventional SM range.

This mechanism can be interpreted as asymptotic freedom, because $\boldsymbol{A_I}/64\pi^2$ determines the strength of quantum corrections in inflationary dynamics [8, 3]. Usually, asymptotic freedom is associated with the asymptotic decrease of some running coupling constants to zero. Here this phenomenon is trickier because it occurs in the interior of the range and fails near its lower and upper boundaries. Quantum effects are small only in the middle part of (37) with a moderately small λ where n_s is close to the "classical" limit $1 - 2/N \simeq 0.967$ for $x \equiv N\boldsymbol{A}/48\pi^2 \ll 1$. Thus, the original claim of [4] on smallness of quantum corrections is right, but this smallness, wherever it takes place, is achieved via a RG summation of big leading logarithms.

Qualitatively our main conclusions are close to those of [5] and [6], though our bounds on the SM Higgs mass are much more sensitive to the CMB data. Therefore, the latter can be considered as a test of the SM theory complimentary to LHC and other collider experiments. The source of this difference from [5, 6] can be ascribed to the gauge and parametrization (conformal frame) dependence of the off-shell effective action (along with the omission of Goldstone modes contribution in [5]) – an issue which is discussed in much detail in [15] and which is expected to be resolved in future publications.

We have also shown the naturalness of the gradient and curvature expansion in this model, which is guaranteed within the conventional perturbation theory range of SM, $\lambda/16\pi^2 \ll 1$, and holds in the whole range of the CMB compatible Higgs mass (37) – the latter property being a consequence of the asymptotic freedom of the above type. This result is achieved by the background field resummation of weak field perturbation theory leading to the replacement of the fundamental Planck mass in the known cutoff $4\pi M_P/\xi$ [26, 27] by the effective one. Partly (modulo corrections to inflaton potential, which are unlikely to spoil its shape) this refutes objections of [26, 27] based on the analysis of scattering amplitudes in EW vacuum background. Smallness of the cutoff in this background does not contradict physical bounds on the Higgs mass originating from CMB data for the following reasons. Determination of M_H of course takes place at the TeV scale much below the non-minimal Higgs cutoff $4\pi M_P/\xi$, whereas inflationary dynamics and CMB formation occur for $\lambda/16\pi^2 \ll 1$ below the *running* cutoff $\Lambda(\varphi) = 4\pi\varphi/\sqrt{\xi}$. It is the phenomenon of inflation which due to exponentially large stretching brings these two scales in touch and allows us to probe the physics of underlying SM by CMB observations at the 500 Mpc wavelength scale.

To summarize, the inflation scenario driven by the SM Higgs boson with a strong non-minimal coupling to curvature looks very promising. This model supports the hypothesis that the appropriately extended Standard Model can be a consistent quantum field theory all the way up to quantum gravity and perhaps explain the fundamentals of all major phenomena in early and late cosmology [39, 40]. Recently, an interesting question about the possibility of a supersymmetric generalizations of the model was considered [41]. In principle, the study of this question can shed a new light on the problem of the range of the validity of the model. Ultimately, it will be the strongly anticipated discovery of the Higgs particle at LHC and the more precise determination of the primordial spectral index n_s by the Planck satellite that might decide the fate of this model.

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Note added

When completing the final version of this work we have received the draft of the paper [42] whose authors kindly informed us on their results prior to publication. They claim that due to asymptotic shift symmetry of the model (or asymptotic scale invariance in the Jordan frame) the field-dependent cutoff at the inflation scale is much higher than (33) and is given by $\Lambda_E \sim \sqrt{\xi}\varphi$, which strongly supports naturalness of Higgs inflation along with its consistency at the reheating and Big Bang stages. The difference from (33) can be explained by the fact that quantum corrections in [42], in contrast to our Jordan frame calculations, were analyzed only in the Einstein frame. In this frame, in particular, the strongest curvature squared counterterm is $O(1)R^2/16\pi^2$ rather than (34) (see [25]). Of course, modulo the conformal anomaly contribution, which only effects the log arguments and cannot be responsible for the onshell discrepancy in these counterterms, the physical results should be equivalent in both parameterizations [15]. This can be an indication of intrinsic cancelations which are not

manifest in the Jordan frame and which could effectively raise the cutoff from its naive value (33) to that of [42]. This would also justify suppression of higher dimensional operators in coefficient functions V, U, G, mentioned in the end of Sect.5. However, verification of frame equivalence of the physical results should be based on gauge and parametrization invariant definition of CMB observables which is currently under study.

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